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Voltage and temperature dependence of current noise in double-barrier normal-superconducting structures

G. Bignon[1], F. Pistolesi[1,a], and M. Houzet[2]

[1] Laboratoire de Physique et Modélisation des Milieux Condensés, Centre Nationale de la Recherche Scientifique and Université Joseph Fourier, B.P. 166 38042 Grenoble, France
[2] Commissariat à l'Énergie Atomique, DSM, Département de Recherche Fondamentale sur la Matière Condensée SPSMS, 38054 Grenoble, France

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Abstract. We study theoretically the current-noise energy (voltage bias and temperature) dependence for a N-N'-S structure, where N and S stand for bulk normal metal and superconductor, respectively, and N' for a short diffusive normal metal. Using quasiclassical theory of current fluctuations we determine the noise for arbitrary distributions of channel transparencies on both junctions. The differential Fano factor turns out to depend on both junction transparencies and the ratio of the two conductances. We discuss analytically the coherent and incoherent regimes and the case when one of the two conductances dominates the other one. Measurement of differential conductance and noise can be used to probe the channel distribution of the interfaces. We discuss recent experiments in the light of our results.

PACS. 73.23.-b Electronic transport in mesoscopic systems – 72.70.+m Noise processes and phenomena – 74.45.+c Proximity effects; Andreev effect; SN and SNS junctions – 74.40.+k Fluctuations (noise, chaos, nonequilibrium sc, localization, etc.)

1 Introduction

Current noise in hybrid mesoscopic systems has been deeply investigated in the last decade, both from the experimental and theoretical side [1,2]. It is quite clear now that noise contains piece of information on the charge transfer mechanism that is not present in the average current. The most striking example is clearly the carriers' elementary charge, that can be obtained by measuring the noise-to-current ratio (Fano factor) in tunnel junctions. As a matter of fact, in mesoscopic Normal metal/Superconducting (N/S) hybrid structures, for energy (voltage bias and temperature) below the superconducting gap, the elementary process responsible for transport is Andreev reflection [3,4]. It involves the transfer of *two* electrons at (nearly) the same time from the superconductor to the normal metal. This implies a *doubling* of the noise that has been predicted [5,6] and observed [7,8]. The situation is particularly clear in the tunneling limit, where the Fano-factor dependence on voltage and noise is exactly that for a normal metal with the replacement $e \to 2e$ [9]. This behavior has been recently observed in semiconductor/Superconductor tunnel junctions [10].

N/S structures are also interesting for another reason. If the mesoscopic structure is shorter than the coherence length, transport is coherent and interference plays a crucial role. Since Andreev reflection involves scattering of an electron and a hole that are nearly time reversed particles, the random phases acquired during the diffusion in the metal are canceled out, and interference between electronic waves is controlled only by the length of the path and the energy of the particles [11]. This leads to a strong energy (temperature or voltage bias) dependence of the conductance that has been predicted [12–14] and measured [15,16]. At large energies, phases acquire a fast dependence on position and transport becomes incoherent.

Very recently, the noise was also shown to have a nontrivial dependence on the energy. This dependence is *different* from that of the conductance [17–20]. The cases of a long diffusive wire [17,21], tunnel junction [9,22], and double tunnel barriers [23] have been considered in the literature.

The last structure is particularly interesting since interference is enhanced by increasing the number of reflections. A Fabry-Perrot structure made of two barriers between the superconductor and the normal metal is expected to show a strong energy dependence of the conductance. This was predicted some time ago [12] for N-I-N'-I-S structures (where I is an insulating barrier) using quasiclassical Green's function approach, and then confirmed experimentally [24–27]. More recently the noise in

[a] e-mail: fabio.pistolesi@grenoble.cnrs.fr

this tunnelling structure has been calculated [23]. The tunnelling condition greatly simplifies the theoretical approach. This assumption does not limit severely the range of the normal-state conductances that can be theoretically investigated since the number of channels in most cases is very large. However, for given normal-state conductances one expects a dependence of current and noise on the actual value of the transparencies. We find actually that this dependence can be the dominant one in some limits. Concerning the current, this was confirmed by the work of Clerk et al. [28] where the conductance for non-tunnel N-N'-S structures has been evaluated by means of random matrix theory. The behavior of the noise when the interfaces are not tunnelling is the object of the present work.

In this paper we calculate the current noise for a N-N'-S structure without restrictions on the distribution of channel transparencies on both interfaces. We use quasiclassical Green's function technique [29–31]. Boundary conditions at the reservoirs are modified by the introduction of a counting field [32–34] which allows to calculate the noise. Exploiting the parametrization for the Green's function proposed by two of the authors in reference [21] we obtain the expressions for the voltage and temperature dependence of current noise in terms of a complex parameter to be found numerically. In some limiting cases the calculation can be performed to the end analytically. In all others the numerics is straightforward. We find that when the conductances are of the same order of magnitude, the channel distribution becomes crucial for the energy dependence of both the current and noise. The expressions we provide can be used to characterize interfaces when current and noise can be measured accurately. Even if this is non trivial from the experimental point of view, one should consider that it is very difficult to control, only by means of the fabrication, the transparency of an interface, *i.e.* the value of the transparencies and their distribution. If the average transparency can be easily estimated from the size of the contact, the true distribution remains out of the reach of any probe. That is why having a theory that predicts the conductance and noise for an arbitrary distribution of the channel transparencies can be a useful tool.

The paper is organized as follows. In Section 2 we introduce the model and derive the main equations. In Section 3 and Section 4 we obtain the expressions for the current and the noise, respectively. Section 4.4 is devoted to the case when the transport is dominated by one interface. Section 5 gives our conclusions.

2 Model and basic equations

We consider a N-N'-S structures with two junctions characterized by their set of channel transparencies: $\{\Gamma_{Nn}\}$ for the N-N' barrier and $\{\Gamma_{Sn'}\}$ for the N'-S barrier, n and n' being channel labels (see Fig. 1). Consequently, the conductances are $g_{N(S)} = g_Q \sum_n \Gamma_{N(S)n}$, where $g_Q = 2e^2/h$ is the quantum of conductance. We assume that $g_{N/S}$ is small enough to completely neglect the voltage drop in the N' part. Namely, we require that the time necessary for an electron incoming from the leads to visit the whole N' region (dwelling time τ_D) is much smaller than the time spent in the region itself (escape time τ). This corresponds to asking that the Thouless energy $E_{\rm Th} \equiv \hbar/\tau_D = \hbar D/L^2$ (D being the diffusive constant and L the typical size of N') is much larger than $E_\tau \equiv \hbar/\tau = (g_N + g_S)\delta/(4\pi g_Q)$ (δ being the average level spacing for N'). We also assume that $L \ll \xi_d = \sqrt{\hbar D/\Delta}$ (or equivalently $E_{\rm Th} \gg \Delta$), where Δ is the superconducting gap of S, so that the spatial dependence of the proximity effect can be neglected in N'.

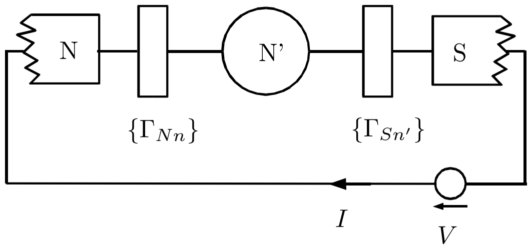

Fig. 1. Schematic picture of the N-N'-S junction. $\{\Gamma_{Nn}\}$ and $\{\Gamma_{Sn'}\}$ are channel transparencies of the N-N' and N'-S barriers.

Proximity effect is thus completely controlled by E_τ and charge transport does not depend on the shape of N'. Hence we can consider N' as an isotropic zero dimensional conductor. We also assume that $g_{N/S} \gg g_Q$: Each barrier has a large number of conduction channels. Coulomb blockade and weak localization effects are then negligible. The upper bound to the conductances is that the notion of chaotic cavity must remain valid, this implies that the surface of the contact must be small with respect the the total external surface of the central island. Finally we require that the escape time is much smaller than phase breaking and inelastic time. All these requirements are met, for instance, in the experiment of reference [26].

Within these assumptions, one can apply the so-called "circuit theory" to calculate current, noise and higher current cumulants [18,33–36]. In particular the central region can be approximated with a single node, since any internal spatial dependence is negligible. The conductor is thus discretized into three nodes connected via two connectors, see Figure 1. Each node is characterized by a quasiclassical matrix Green's function in the Keldysh($\bar{\ }$)-Nambu($\hat{\ }$) space, $\check{G}_{N/S}$ for N and S leads and $\check{G}$ for N' depending on the energy E and a counting field χ [32].

The counting field appears as a modification of the boundary conditions. In our case this corresponds to transforming the normal reservoir Green's function as follows [34]:

$$\check{G}_N(\chi) = e^{i\chi\check{\tau}_K/2}\ \check{G}_{N0}\ e^{-i\chi\check{\tau}_K/2}, \tag{1}$$

where $\bar{\sigma}_i, \hat{\tau}_{j(i,j=1,2,3)}$ are Pauli matrices, $\check{\tau}_K = \hat{\tau}_3 \otimes \bar{\sigma}_1$, and $\check{G}_{N0}$ is the normal metal quasiclassical Green's function in the diffusive limit (for a recent review see Ref. [37]):

$$\check{G}_{N0} = \begin{pmatrix} \hat{\tau}_3 & 2(f_{T0} + f_{L0}\hat{\tau}_3) \\ 0 & -\hat{\tau}_3 \end{pmatrix}. \quad (2)$$

Here, $f_{T0} = f(E - eV) - f(E + eV)$, $f_{L0} = 1 - f(E + eV) - f(E - eV)$, f is the Fermi function at temperature T, and V is the voltage bias between the normal metal and the superconducting reservoir.

The Green's function in the superconducting reservoir is

$$\check{G}_S = \begin{pmatrix} \hat{R}_S & \hat{K}_S \\ 0 & \hat{A}_S \end{pmatrix}. \quad (3)$$

Here, $\hat{R}_S$ is the retarded part given by:

$$\hat{R}_S = \frac{1}{\sqrt{(E + i\eta)^2 - |\Delta|^2}} \begin{pmatrix} E & \Delta \\ -\Delta^\star & -E \end{pmatrix} \quad (4)$$

with the branch cut of the square root going from $-|\Delta|$ to $+|\Delta|$ on the real E axis. The advanced part is given by $\hat{A}_S = -\hat{\sigma}_3 \hat{R}_S^\dagger \hat{\sigma}_3$, and the Keldysh part follows by the equilibrium condition of the reservoir: $\hat{K}_S = (f(E) - f(-E))(\hat{A}_S - \hat{R}_S)$. In the following, we focus on the subgap regime, so we can limit ourselves to $E \ll |\Delta|$. Moreover, since there is only one superconductor in the problem, we can choose Δ real. Then the matrix Green's function of the superconductor simplifies to $\check{G}_S = \hat{\tau}_2 \otimes \bar{1}$.

The Green's function in the central node satisfies the normalization condition $\check{G}^2 = \check{1}$ and the symmetry property [21,38]:

$$\check{G}^\dagger(-\chi) = -\check{\tau}_L \, \check{G}(\chi) \, \check{\tau}_L \quad (5)$$

with $\check{\tau}_L = \hat{\tau}_3 \otimes \bar{\sigma}_2$. (Similar relations hold for $\check{G}_{N/S}$ as well.) It is solution of the Usadel equation [31,37]:

$$\hbar D \boldsymbol{\nabla} \left(\check{G} \boldsymbol{\nabla} \check{G}\right) + iE[\check{G}_E, \check{G}] = 0\,, \qquad \check{G}_E = \hat{\tau}_3 \otimes \bar{1}\,. \quad (6)$$

We integrate this equation over the volume $\mathcal{V}$ of N'. Using the divergence theorem, it gives:

$$\int_{\partial\mathcal{V}} d^2\mathbf{S} \cdot \left(\sigma_0 \check{G} \boldsymbol{\nabla} \check{G}\right) + 2\,i \frac{e^2 \nu_0 \mathcal{V} E}{\hbar} [\check{G}_E, \check{G}] = 0\,, \quad (7)$$

where ν_0 is the density of states per spin of N', and $\sigma_0 = 2e^2 D \nu_0$ its conductivity in the normal metallic state. Using boundary conditions for the Green's functions over the surface ∂V of the grain [39,35], we have:

$$-\int_{\partial\mathcal{V}} d^2\mathbf{S}.\left(\sigma_0 \check{G} \boldsymbol{\nabla} \check{G}\right) = \check{I}_N + \check{I}_S \quad (8)$$

with

$$\check{I}_N = g_Q \sum_n \frac{2\,\Gamma_{Nn} [\check{G}_N(\chi), \check{G}(\chi)]}{4 + \Gamma_{Nn}(\{\check{G}_N(\chi), \check{G}(\chi)\} - 2)}, \quad (9)$$

$$\check{I}_S = g_Q \sum_n \frac{2\,\Gamma_{Sn} [\check{G}_S, \check{G}(\chi)]}{4 + \Gamma_{Sn}(\{\check{G}_S, \check{G}(\chi)\} - 2)}\,. \quad (10)$$

Then $\check{G}$ is fully determined by equation (6) which takes the form of a conservation-like equation for the spectral matrix current:

$$\check{I}_N + \check{I}_S + \check{I}_E = 0\,, \quad (11)$$

where

$$\check{I}_E = -g_Q \frac{2i\pi\, E}{\delta} [\check{G}_E, \check{G}(\chi)]\,. \quad (12)$$

Here $\check{I}_E$ is the "leakage" matrix current [35], which takes into account the relative dephasing between electron and hole during their propagation in the central node N', whose mean level spacing is $\delta = 1/(\nu_0 \mathcal{V})$. The estimate for the inverse escape time, $E_\tau/\hbar = \delta(g_N + g_S)/(4\pi\hbar\, g_Q)$, follows from comparison between the amplitudes of $\check{I}_N + \check{I}_S$ and $\check{I}_E$.

Once the matrix $\check{G}(\chi)$ is known, current, zero frequency noise and all higher current cumulants can be obtained by differentiation of $I(\chi)$ defined as follows:

$$I(\chi) = -\frac{1}{8e} \int \mathrm{d}E \; \mathrm{tr}[\check{\tau}_K \check{I}_N]. \quad (13)$$

(By matrix current conservation (11) $I(\chi)$ equals minus expression (13) with $\check{I}_N$ substituted by $\check{I}_S$.) The first two moments are the average current,

$$I = I(\chi)\big|_{\chi=0}, \quad (14)$$

and the current noise,

$$S = 2ie \left. \frac{\partial I(\chi)}{\partial \chi} \right|_{\chi=0}. \quad (15)$$

For tunneling interfaces, $\Gamma_n \ll 1$, the boundary conditions simplifies since one can neglect the anticommutator in the denominator of equations (9) and (10). In that limit the matrix $\check{G}(\chi)$ can be found analytically [23,40].

It is thus possible to study not only the current and noise, but the whole set of cumulants. In the general case of arbitrary value of Γ_n there is no analytical solution available for $\check{G}(\chi)$.

If one restricts to the first two cumulants, I and S, which are more accessible experimentally, it is possible to write simplified equations for the coefficient of the expansion of $\check{G}$ in χ [17,21]:

$$\check{G}(\chi) = \check{G}_0 - i\frac{\chi}{2}\check{G}_1 + \mathcal{O}(\chi^2). \quad (16)$$

Finding $\check{G}_0$ gives the current while $\check{G}_1$ leads to the noise. In the following we follow this program and solve (11) for the first two orders in χ.

3 Current

To obtain the current one has to evaluate equation (13). For this, we need $\check{G}_0$ as defined in equation (16). A crucial step to solve the problem is to take into account the normalization condition without redundancy in the parametrization. When the counting field vanishes, the solution is well known and it consists in the following parametrization of $\check{G}_0$:

$$\check{G}_0 = \begin{pmatrix} \hat{R} & \hat{K} \\ 0 & \hat{A} \end{pmatrix} \tag{17}$$

with

$$\hat{R} = \hat{\tau}_3 \cosh\theta + i\hat{\tau}_2 \sinh\theta\,, \quad \hat{A} = -\hat{\tau}_3\ \hat{R}^\dagger\ \hat{\tau}_3\,, \tag{18}$$

$$\hat{K} = \hat{R}\hat{f} - \hat{f}\hat{A}, \qquad \hat{f} = f_L + \hat{\tau}_3 f_T\,. \tag{19}$$

Here, the parameters f_T and f_L are real, as follows from equation (5) at $\chi = 0$. The complex number θ characterizes the pairing in the grain: Im $\theta = -\pi/2$ corresponds to a BCS superconducting state and Im $\theta = 0$ to a normal one. Substituting this form for $\check{G}_0$ into equation (11) at $\chi = 0$ one can determine θ, f_L, and f_T. The retarded or advanced parts give the equation for θ:

$$g_N\langle Z_N\rangle \sinh\theta - i\varepsilon g_D \sinh\theta + i g_S\langle Z_S\rangle \cosh\theta = 0, \tag{20}$$

where $\varepsilon = E/E_\tau$, $g_D = g_N + g_S$, $Z_N = [1 + \Gamma_N(\cosh\theta - 1)/2]^{-1}$ and $Z_S = [1 + \Gamma_S(i\sinh\theta - 1)/2]^{-1}$. Here,

$$\langle f(\Gamma_\alpha)\rangle = \frac{\sum_n \Gamma_{\alpha n} f(\Gamma_{\alpha n})}{\sum_n \Gamma_{\alpha n}} \tag{21}$$

stands for the average over channel transparencies with α = N or S. The Keldysh part of the spectral-current-conservation equation (11) gives $f_L = f_{L0}$ and $f_T/f_{T0} = c$ with

$$c = -\frac{g_N \tanh\theta_1}{2\,\varepsilon g_D \sin\theta_2}\langle[(2-\Gamma_N)\,\cos\theta_2 + \Gamma_N\,\cosh\theta_1]\,|Z_N|^2\rangle. \tag{22}$$

Here, we used the decomposition $\theta = \theta_1 + i\theta_2$ into real and imaginary part. Finally, the mean current is given by

$$I = \frac{1}{2e}\int_{-\infty}^{\infty} \mathrm{d}E\, f_{T0}\,\mathcal{G}(E) \tag{23}$$

with

$$\mathcal{G}(E) = c\, g_S\, \cosh\theta_1 \times\langle[-\sin\theta_2 + \Gamma_S\,(\cosh\theta_1\, + \,\sin\theta_2)/2])|Z_S|^2\rangle\,. \tag{24}$$

At zero temperature the differential conductance $G \equiv \mathrm{d}I/\mathrm{d}V$ equals $\mathcal{G}(eV)$. For uniform transparency, expression (24) coincides with that obtained by Clerk et al. in reference [28] using random matrix theory. When the conductance of the central island cannot be neglected and $\Gamma_N \ll 1$, the conductance has been considered in reference [21,41].

We now discuss the conductance for small and large energy.

3.1 Coherent regime

Let us begin with the low energy limit $eV \ll E_\tau$, i.e., the completely coherent case. θ is then pure imaginary. Equation 24 reduces to:

$$G_{\rm coh}^{-1} = (\widetilde{g}_N)^{-1} + (\widetilde{g}_S)^{-1}\,, \tag{25}$$

where

$$\widetilde{g}_N = g_N \cos\alpha\langle\widetilde{Z}_N^2\rangle + g_N\langle\Gamma_N\widetilde{Z}_N^2\rangle(1-\cos\alpha)/2\,,$$
$$\widetilde{g}_S = g_S \sin\alpha\langle\widetilde{Z}_S^2\rangle + g_S\langle\Gamma_S\widetilde{Z}_S^2\rangle(1-\sin\alpha)/2\,,$$

with $\widetilde{Z}_N = [1 + \Gamma_N(\cos\alpha - 1)/2]^{-1}$ and $\widetilde{Z}_S = [1 + \Gamma_S(\sin\alpha - 1)/2]^{-1}$. The real parameter $\alpha = -\operatorname{Im}(\theta)$ is the solution of the equation:

$$g_N\,\sin\alpha\,\langle\widetilde{Z}_N\rangle = g_S\,\cos\alpha\,\langle\widetilde{Z}_S\rangle\,. \tag{26}$$

Coherent conductance strongly depends on the ratio g_N/g_S. When the central island is well connected to N ($g_N \gg g_S$), the solution of equation (26) is $\theta = 0$. The grain is in the normal state. Then differential conductance is given by $G_{\rm coh} = g_S^{\rm And} = 2g_S\langle\Gamma_S/(2-\Gamma_S)^2\rangle$: the charge transfer is dominated by Andreev reflection at N'-S interface [42]. In the opposite case of an island well connected to S ($g_N \ll g_S$) the solution is $\theta = -i\pi/2$, the grain is superconducting and we have $G_{\rm coh} = g_N^{\rm And} = 2g_N\langle\Gamma_N/(2-\Gamma_N)^2\rangle$. This means that conductance is dominated by Andreev reflection at N-N' barrier. We can also note that $G_{\rm coh}$ is invariant under the transformation $\{\Gamma_{Nn}\} \leftrightarrow \{\Gamma_{Sn'}\}$ in equation (25). Thus when an electron crosses the N-N'-S structure, it cannot distinguish which barrier is closer to the superconductor. It is interesting to notice that even if transport properties ($G_{\rm coh}$ and $F_{\rm coh}$) do not depend on the relative position of the barriers, the state of the grain does. It can be normal or fully superconducting depending on g_N/g_S.

3.2 Incoherent regime

In the opposite limit of $eV \gg E_\tau$ transport is incoherent. The large energy mismatch between electrons and Andreev reflected holes washes out interference effects. We find the following expression for the conductance

$$G_{\rm class}^{-1} = g_N^{-1} + (g_S^{\rm And})^{-1}\,. \tag{27}$$

Equation (27) is now no more invariant for exchange of the N-S and N'-S barriers. The grain is in the normal state ($\theta = 0$). The physical interpretation for the incoherent transport is simple since one can treat one channel at the time (electrons do not interfere) [7,43]. For a Cooper pair to be transferred across the double barrier structure the electron has to undergo the following steps: crossing of the N-N' barrier, Andreev conversion to a hole at the N'-S junction (with probability $R_{Sn'} = \Gamma_{Sn'}^2/(2-\Gamma_{Sn'})^2$ per channel), and finally crossing of the N-N' barrier (see Fig. 2) [44]. Thus in the incoherent

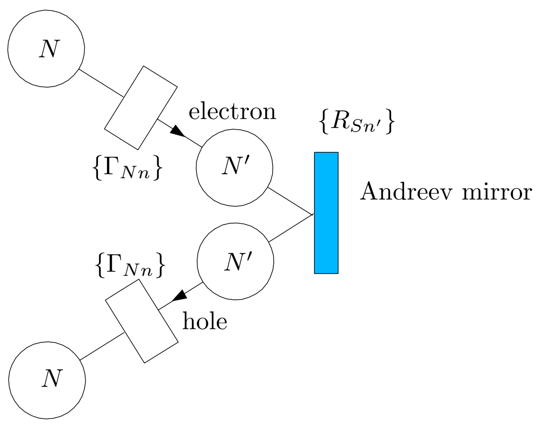

Fig. 2. Schematic picture of the N-N'-S junction in the incoherent regime. Electrons are Andreev reflected into holes at N'-S barrier.

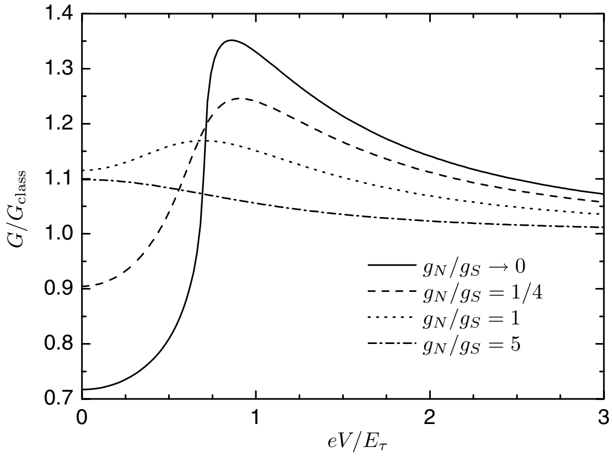

Fig. 3. Differential conductance normalized by its classical value as a function of eV/E_τ for two disordered interfaces at zero temperature. A peak appears near $eV \approx E_\tau$ when $g_N \approx g_S$ and it becomes sharper for $g_N/g_S \to 0$.

limit, the double junction is equivalent to three junctions in series of transparencies $\{\Gamma_{Nn}\}$, $\{R_{Sn'}\}$ and $\{\Gamma_{Nn}\}$, respectively, with an elementary transferred charge $2e$. Conductance is then given by Ohm's law for the three conductances in series multiplied by a factor two: $G_{\rm class} = 2[(g_Q \sum_n \Gamma_{Nn})^{-1} + (g_Q \sum_{n'} R_{Sn'})^{-1} + (g_Q \sum_n \Gamma_{Nn})^{-1}]^{-1}$, which coincides with equation (27).

3.3 Intermediate energies

For intermediate energies, the shape of G depends both on the ratio of the two conductances and the set of channel transparencies. A particularly relevant situation is that of a disordered interface which may form between two metals in the absence of oxide barriers. In such case the channel distribution was shown to be universal [45]:

$$\rho_\alpha(\Gamma) \equiv \sum_n \delta(\Gamma - \Gamma_{\alpha n}) = \frac{g_\alpha}{g_Q\, \pi} \frac{1}{\Gamma^{3/2}\sqrt{1-\Gamma}}\,, \qquad (28)$$

where α =N or S. We plot the conductance for this case and different values of the ratio g_S/g_N in Figure 3. Qualitatively one sees a cross-over from a "reflectionless" tunneling behavior, typical of tunnel junctions (with a zero-bias peak) to a "re-entrant" behavior with a peak at eV of the order of E_τ. In both cases the qualitative explanation is simple. In the tunnel case, the electron tries many times to enter the superconductor. At low energy, the corresponding quantum paths add coherently, giving a large resulting current. Any increase in the energy reduces the coherent contribution to the current since interference is suppressed and, thus, the mixed terms vanish. This explains the zero-bias peak in the $G(V)$ plot. On the other hand, when the superconducting barrier is transparent the electrons always succeed in being converted to holes, but Andreev reflection comes with a phase factor $(-i)$ that induces destructive interference among electronic waves for $E = 0$ [46]. The loss of coherence among waves can thus enhance the current leading to a maximum in the $G(V)$ plot. This behavior is very similar to the one observed in a diffusive wire [12]. One sees nevertheless that the effect is much larger here, since the Fabry-Perot structure enhances interference. We will discuss the role of barrier transparencies in Section 4.3.

4 Noise

For intermediate values of the conductances the noise is a true non-equilibrium property, and cannot be obtained from the knowledge of the physical ($\chi = 0$) Green's function alone. As stated above we need to solve equation (11) in first order in χ. We expand thus each spectral current: $\check{I}_\beta = \check{I}^0_\beta + \chi \check{I}^1_\beta + \mathcal{O}(\chi^2)$ with $\beta =$ N, S, or E. We obtain:

$$\begin{aligned}\check{I}^1_N &= i g_N \big(\langle \check{D}_N\rangle ([\check{G}_{N1}, \check{G}_0] - [\check{G}_{N0}, \check{G}_1]) \\ &\quad - \langle \Gamma_N [\check{G}_{N0}, \check{G}_0] \check{D}_N (\{\check{G}_{N1}, \check{G}_0\} - \{\check{G}_{N0}, \check{G}_1\}) \check{D}_N\rangle\big)\,, \\ \check{I}^1_S &= -i g_S \big(\langle \check{D}_S\rangle [\check{G}_S, \check{G}_1] \\ &\quad - \langle \Gamma_S [\check{G}_S, \check{G}_0] \check{D}_S \{\check{G}_S, \check{G}_1\} \check{D}_S\rangle\big)\,, \\ \check{I}^1_E &= -\frac{g_D \varepsilon}{4} [\check{G}_E, \check{G}_1]\,,\end{aligned}$$

with $\check{D}_\alpha = (4 + \Gamma_\alpha(\{\check{G}_{\alpha 0}, \check{G}_0\} - 2))^{-1}$, α =N or S, and $\check{G}_{N1} = [\check{\tau}_K, \check{G}_{N0}]$. Zero-frequency current noise is given by

$$S = \frac{i}{4} \int \mathrm{d}E\ \mathrm{tr}[\check{\tau}_K \check{I}^1_S]\,. \qquad (29)$$

Here, the unknown matrix $\check{G}_1$, cf. equation (16), satisfies

$$\check{I}^1_N + \check{I}^1_S + \check{I}^1_E = 0\,. \qquad (30)$$

Additionally the normalization of $\check{G}$ implies $\{\check{G}_0, \check{G}_1\} = 0$. This can be satisfied by defining $\check{G}_1 = [\check{G}_0, \check{\phi}]$ for any $\check{\phi}$. We use the parametrization found in reference [21] for the matrix $\check{\phi}$:

$$\check{\phi} = \begin{pmatrix} a f_{T0} \hat{\tau}_1 - c \hat{f} \hat{\tau}_3 & b \hat{\tau}_3 + d \\ c \hat{\tau}_3 & a^* f_{T0} \hat{\tau}_1 + c \hat{f} \hat{\tau}_3 \end{pmatrix}\,. \qquad (31)$$

The symmetry condition (5) on $\check{G}$ implies that $\check{\phi}^\dagger = -\check{\tau}_L\check{\phi}\check{\tau}_L$; it follows that b, c, and d are real, while a is complex. The parameter c has been already given in equation (22). Inserting this form for $\check{\phi}$ into equation (30) one obtains a complete set of equations for all the parameters of $\check{\phi}$. The equation for a is given by the antidiagonal elements of the retarded part of equation (30). The Keldysh part of the same equation gives the equations for b and d. Finally, using equation (29), zero frequency current noise takes the form:

$$S = \int \mathrm{d}E \, \{\mathcal{G}(E)[1 - f_{L0}^2] + \mathcal{S}_T(E)\, f_{T0}^2\}\,. \tag{32}$$

The rather cumbersome expressions for a, b, d, and $\mathcal{S}_T$ are given in Appendix A. Here we only stress that the analytic expressions for the coefficients all depend on a single complex number, θ, solution of equation (20). Even if θ is given by the solution of an algebraic equation it is not always possible to obtain an analytical expression for it. Nevertheless, once this parameter is known numerically, it is enough to substitute it into the expressions given in Appendix A to obtain the value of the current noise. Note that knowledge of θ is already necessary to obtain the conductance.

Let us now discuss the result in some details. We first note that equation (32) for $eV \ll k_BT$ correctly agrees with the fluctuation-dissipation theorem [47]. As a matter of fact, in this case $f_T = 0$ and the remainder gives precisely $S = 4k_BT\,G(T)$. In the opposite limit noise is not simply related to the conductance and has to be computed with equation (32). In the zero temperature limit ($k_BT \ll eV$, E_τ) the experimentally accessible differential Fano factor becomes from equation (32):

$$F(V) \equiv \frac{1}{2eG(V)}\frac{\mathrm{d}S(V)}{\mathrm{d}V} = 1 + \frac{\mathcal{S}_T(eV)}{\mathcal{G}(eV)}\,. \tag{33}$$

Let us now discuss as in the conductance case the two analytically tractable limits: the completely coherent and incoherent cases.

4.1 Coherent regime

In the coherent limit one can obtain closed analytical expression for the noise depending on the parameter α solution of equation (26). However they are rather cumbersome and we will not show them. In the specific case of two transparent barriers, we recover the recent analytical result of Vanević et al. in reference [48]. Similarly to the conductance, the expression for the Fano factor is left unchanged when the set of transparencies of the two barriers are exchanged: $\{\Gamma_{Nn}\} \leftrightarrow \{\Gamma_{Sn'}\}$. The Fano factor depends on the ratio g_N/g_S. If the grain is well connected to the superconductor ($g_N \ll g_S$), $F_{\mathrm{coh}} = 2\sum_{n'} R_{Sn'}(1 - R_{Sn'})/\sum_{n'} R_{Sn'}$: we obtain the Fano factor of N'-S interface alone [6]. In the opposite limit, $g_N \gg g_S$, $F_{\mathrm{coh}} = 2\sum_n R_{Nn}(1 - R_{Nn})/\sum_n R_{Nn}$: the Andreev reflection occurs at N'-N barrier.

4.2 Incoherent regime

We consider now the incoherent limit: $eV \gg E_\tau$. From equation (33) we find the following explicit form for the differential Fano factor

$$F_{\mathrm{class}} = \Bigg[(g_S^{\mathrm{And}})^3\frac{\sum_n \Gamma_{Nn}(1-\Gamma_{Nn}/2)}{\sum_n \Gamma_{Nn}} + 2g_N g_S^{\mathrm{And}}(g_N + g_S^{\mathrm{And}}) + 2g_N^3\frac{\sum_n R_{Sn}\,(1 - R_{Sn})}{\sum_n R_{Sn}}\Bigg]\frac{1}{(g_N + g_S^{\mathrm{And}})^3}. \tag{34}$$

This result can also be found using the technique developed by Belzig et al. in reference [49]. The physical interpretation is the same described for G_{class}, the only difference is that here we need to calculate the current fluctuation at each barrier instead of the current. Indeed, in the classical limit, the structures can be schematized as a series of three junctions of transparencies $\{\Gamma_{Nn}\}$, $\{R_{Sn'}\}$, and $\{\Gamma_{Nn}\}$ with decoherent cavities in between. Again the elementary transferred charge is $2e$ (see Fig. 2). The Fano factor for a series of two junctions separated by a decoherent cavity has been evaluated (for elementary charge e) [18,50]:

$$F_{12}(g_1, F_1; g_2, F_2) = \frac{g_1^3F_2 + g_1g_2(g_1 + g_2) + g_2^3F_1}{(g_1 + g_2)^3}, \tag{35}$$

where $g_i = g_Q\sum_n \Gamma_{in}$ and $F_i = (\sum_n \Gamma_{in}(1 - \Gamma_{in}))/\sum_n \Gamma_{in}$, $i = 1$ or 2. From equation (35) the Fano factor for three junctions in series can be easily obtained: $F_{123}(g_1, F_1; g_2, F_2; g_3, F_2) = F_{12}(g_{12}, F_{12}; g_3, F_3)$ with $g_{12} = g_1g_1/(g_1 + g_2)$. This expression coincides with equation (34), once we take into account the doubling of the charge. Let us now consider the case when one of the two interfaces dominates transport. For $g_S^{\mathrm{And}} \ll g_N$, N'-S junctions controls charge transfer and it is thus not surprising to find that the Fano factor is that of the N-S barrier alone [6]: $F_{\mathrm{class}} = 2\sum_n R_{Sn}\,(1 - R_{Sn})/\sum_n R_{Sn}$. In the opposite limit of $g_S^{\mathrm{And}} \gg g_N$, we have instead the following result: $F_{\mathrm{class}} = \sum_n \Gamma_{Nn}\,(1 - \Gamma_{Nn}/2)/\sum_n \Gamma_{Nn}$. Note that it differs from the Fano factor for a single interface of transparency distribution Γ_{Nn}. Actually even if the resistance is dominated by the N'-N interface, the presence of the N-S interface doubles the number of interfaces, leading to this result. Note also that for a completely transparent N'-N interface we have a finite noise $F = 1/2$. The conductance in this limit is g_N (cf. Eq. (27)). Again one could expect that F should be zero, but actually transport is slightly more subtle. The effective system is that of a chaotic cavity connected through two completely transparent interfaces of conductance g_N to the two leads. The electron entering the cavity from the normal side has probability 1/2 of exiting from the same interface as an electron and 1/2 of exiting as a hole on the other side. In the second case the transferred charge is $2e$ with probability $\Gamma = 1/2$. Thus the effective conductance is g_N, like for a normal Sharvin contact, but with an effective Fano factor of 2 (for the charge) times 1/4 (for the $\Gamma(1 - \Gamma)$ term).

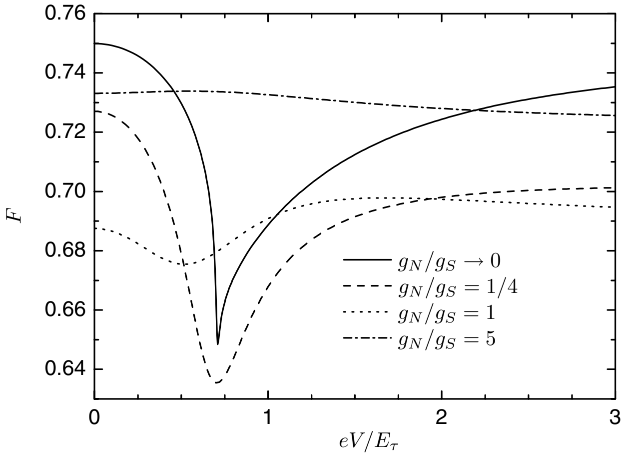

Fig. 4. Differential Fano factor as a function of eV/E_τ for two disordered interfaces at zero temperature. A dip appears for $eV \approx E_\tau$ when $g_N \approx g_S$ and increases for g_N/g_S decrease.

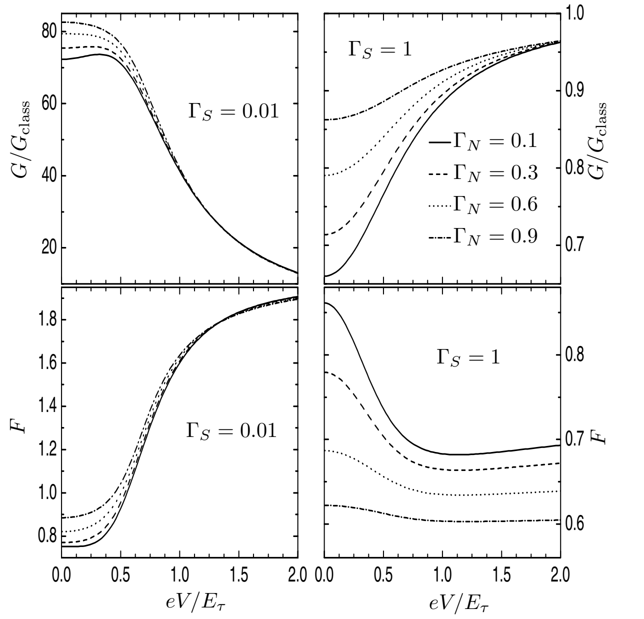

Fig. 5. Differential conductance and Fano factor as a function of eV/E_τ at zero temperature. Channel transparencies of the two interfaces have unimodal distribution. In the left panel, N'-S junction is tunnel ($\Gamma_S = 0.01$) and in the right panel N'-S junction is transparent ($\Gamma_S = 1$).

4.3 Intermediate energies

For intermediate values of the energy, noise, like the conductance, has to be considered numerically. Our results allow to study any situation. We plot in Figure 4 the Fano factors for the same parameters considered for the conductance in Figure 3. The qualitative behavior resembles that of the noise in long diffusive structures, but with a stronger energy dependence. In particular a minimum at finite voltage for the Fano factor is present when $g_S \gg g_N$. This is very similar to the minimum in the differential Fano factor for a wire in good contact with normal and superconducting reservoirs [8,36].

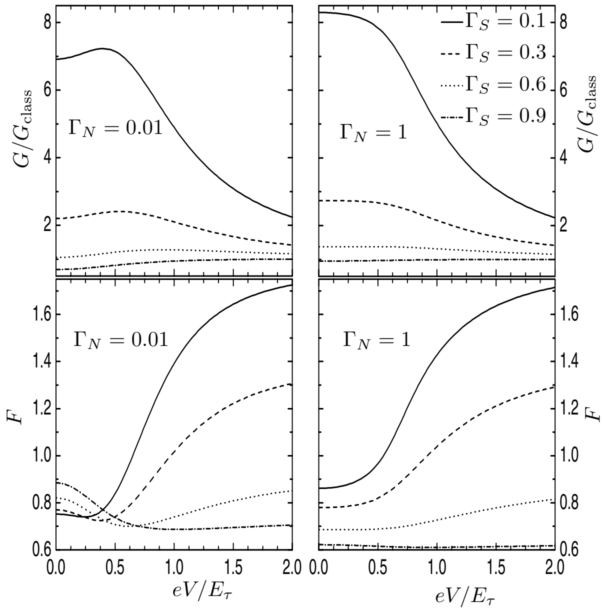

Fig. 6. Differential conductance and Fano factor as a function of eV/E_τ at zero temperature. Channel transparencies of the two interfaces have unimodal distribution. In the left panel, N-N' junction is tunnel ($\Gamma_N = 0.01$) and in the right panel N-N' junction is transparent ($\Gamma_N = 1$).

Let us now consider the genuine effect of the modification of the channel distribution. The optimal situation is when the normal-state conductances of the two interfaces are equal, so that the dependence on the distribution of the channel transparencies of both interface should be maximum. For simplicity in the presentation we will discuss only the case of $\Gamma_n = \Gamma$ for all n. We thus vary the transparency and the number of channels at each interface in such a way that the ratio of the normal-state conductance is kept equal to 1. Note when the channels are transparent this does not mean necessary that the contact region must be very small (to keep the same conductance of the tunneling case). It is enough that the distribution of channels of a large junction is bimodal, with a large majority of the channels completely opaque ($\Gamma = 0$) and few of them of the given transparency.

We calculated the energy dependence of the conductance and of the noise as a function of the transparency of the interfaces. Results are reported in Figure 5 and Figure 6. In Figure 5 we set $\Gamma_S = 0.01$ (left pannels) and $\Gamma_S = 1$ (right pannels) and we vary Γ_N from 0.1 to 0.9. In Figure 6 we plot the same curves exchanging the role of Γ_N and Γ_S.

First we note how important is the channel transparency to predict the value of both the conductance and the noise. Knowledge of the conductance alone is not enough. Once the conductance is known, the energy dependence of both current and noise can give valuable indications on the channel distribution. A second important remark is the qualitatively similar behavior of the conductance and the Fano factor. This is particularly evident for the case when $\Gamma_N \gg \Gamma_S$ (left panel of Fig. 5).

The differential Fano factor is linearly related to the conductance, $F(E) \approx \gamma_0 - \gamma_1 G(E)$, with γ_0 and γ_1 positive constants. This behavior was proved analytically for tunneling contacts between a superconductor and a wire in reference [21]. Actually this behavior seems to be a general property of the whole set of plots, with variable accuracy. The differential Fano factor looks like the differential conductance upside down. This is only a qualitative behavior, the proportionality factor depends on the actual transparency, as was found in reference [21].

4.4 Current fluctuations for g_S/g_N large or small

When one of the two conductances dominates the other, the transport (current and current fluctuations) should be completely determined by the interface with the smaller conductance. This is only partially true. Let us discuss the case $g_N/g_S \to 0$. By dividing equation (11) by g_S, $\check{I}_N$ drops from the equation. The central-node Green's function becomes then independent of the N contact, and, in particular, it is independent of its counting field χ. The Green's function can thus be completely parameterized by θ that satisfies equation (20) (since $f_L = f_{L0}$ and f_T is expressed in terms of θ.) Once the value of θ is known, current fluctuations can be obtained by inserting $\check{G}(\theta)$ into equation (9) and evaluating equation (13). This gives the generating function for the current fluctuations [34]

$$H(\chi) = -\sum_n \int dE \,\mathrm{tr}\ln\left[1 + \frac{\Gamma_{Nn}}{4}(\{\check{G}_N(\chi), \check{G}(\theta)\} - 2)\right] \quad (36)$$

defined by $I = -ig_Q/(4e)\,\partial H/\partial\chi$. The current and the noise are readily obtain from equations (14) and (15).

Let us now consider the role of Γ_S. For $g_N/g_S \to 0$ the equation for θ simplifies to:

$$\left\langle \frac{1}{1 + \Gamma_S(i\sinh\theta - 1)/2} \right\rangle \cosh\theta - \varepsilon\sinh\theta = 0\,. \quad (37)$$

If $\Gamma_{Sn} \ll 1$ one thus recovers the BCS Green's function with an effective gap equal to E_τ. This implies that the current and the noise will be given by the Blonder-Tinkham-Klapwik [3] and Khlus [5] expressions for a single S-N barrier, respectively. For non tunnelling Γ_{Sn} the wavefunction of the island differs from the BCS one. The current and noise in this case can then be readily obtained from expression (36), once the channel distribution on the two interfaces is given. As anticipated, charge transport actually depends on Γ_S even if $g_N/g_S = 0$. The reason is the influence of Γ_S on the wave function of the central island. This effect was first recognized by Melsen et al. by looking at the density of states of a chaotic cavity in contact with a superconductor [51].

It is also interesting to discuss the condition of large g_S. It is clear that the condition $g_S \gg g_N$ suffices to drop g_N from the equation for θ. But the same condition does not guarantee that the current-fluctuations are dominated by the N/N' interface. From equation (27) for the conductance in the incoherent regime it turns out that the actual condition is $g_S^A \gg g_N$. In the tunnelling regime this gives $g_S \gg g_S^A = g_S\,\langle\Gamma_S\rangle \gg g_N$. One finds $F = 2$ for $\varepsilon < 1$, when current is dominated by tunneling of Cooper pairs, and $F = 1$ for $\varepsilon > 1$, when current is dominated by quasiparticle tunneling.

If one takes the limit $\Gamma_S \to 0$ first, then the $g_S^A \gg g_N$ condition can never be satisfied. That's why calculations performed with Kuprianov-Luckichev tunneling boundary conditions give a different result for the Fano factor in the $g_N/g_S \to 0$ limit. Namely the limit $g_1/g_2 \to 0$ of equation (28) of reference [23] gives the following expression for the differential Fano factor: $F(\varepsilon) = 2$ for $\varepsilon < 1$ and $F(\varepsilon) = 2 - 1/\varepsilon^2$ for $\varepsilon > 1$. Both calculations concern tunnelling interfaces, but in reference [23] the limit $g_S^A = \langle\Gamma_S\rangle g_S \ll g_N \ll g_S$ is implicitly taken. In the next section we shall discuss this result in connection to a recent experiment.

For $g_N \gg g_S$ a similar analysis can be performed leading to the full current fluctuations. The result is more trivial, because θ vanishes identically in this limit. Thus the resulting current fluctuations are those of a normal metal in contact with a superconductor: The central island is driven to the normal state for any value of the N/N' transparency.

5 Discussion and conclusions

We have studied the energy dependence of the current noise in a double barrier N-N'-S structure for arbitrary transparency of the barriers. We provided explicit expressions in terms of a single complex number, θ, to be determined numerically. We described analytically the following limits: g_N/g_S large or small, the completely coherent ($eV \ll E_\tau$) and completely incoherent ($eV \gg E_\tau$) case. The full energy dependence for different values of the transparencies has been studied numerically for $g_S = g_N$. From our analysis we can draw some general conclusions: The noise in the double-barrier structure has a much more pronounced energy dependence than previously studied noise in extended geometry, like a wire [21]. We find that the energy dependence of the current and noise are qualitatively related though quantitatively independent. The distribution of transparencies at the barriers plays an important role in the determination of both the current and noise. As shown in Figures 5 and 6 it determines the qualitative behavior.

Let us now discuss recent experimental results [10,26]. It is generally found that the quasiclassical theory gives a qualitatively good agreement with experimental energy dependence of the conductance, even if a quantitative comparison is in general very difficult. When heating or Coulomb blockade effects can be neglected, the main problem is guessing the true channel distribution of the interface. Let us for instance consider the experiment [26] where the conductance for the structure considered in this paper has been measured for $g_S \approx g_N$ and for $g_S \gg g_N$. The fabrication procedure is expected to lead to $\Gamma_N, \Gamma_S \ll 1$ in the first case, and actually the agreement with reference [12] is reasonably good. In the second case, the distribution of Γ_S is not known, but a dominance of transparent channel is expected, since on the same surface the conductance is much larger. Actually the authors found that it was not possible to fit this data with Volkov theory, but at the same time the presence of a peak at finite energy should indicate that Γ_S are not really near 1. It is difficult to try a fit of the conductance alone, when too many parameters are not known. But a measurement of the noise in the same sample could be possible. Note that this configuration is probably ideal since the double barrier structure keep the current low avoiding heating effects. The joint measurement of the differential conductance and Fano factor could thus give a definite information on the transparency and allow a quantitative interpretation of the data.

This is the case for other experimental result by the same group [10]. They measured the conductance and the noise in a tunnelling semiconductor/superconductor structure. They observed a transition of the differential Fano factor from 2 to 1 by increasing the voltage. What is still not understood is that the crossing value was not the superconducting gap, but a much smaller energy scale. A possible explanation of this effect come from our theory. The rather complex interface could be modelled as a double barrier, where an intermediate normal region is well connected to the superconductor and is connected to the normal metal through the Schottky tunnelling contact. In that case the discussion of Section 4.4 for $g_N \ll g_S^A \ll g_S$ applies and the observed behavior can be interpreted as the crossover of F from 2 to 1 at the energy scale E_τ, smaller than the superconducting gap scale. Moreover, the fact that $F = 2$, or 1, is a strong indication that the interface is dominated by small values of the transparency.

Similar measurements could shed more light on the nature of interfaces and allow a better understanding of the transport in superconducting/normal metal hybrid structures.

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Appendix A: Fano factor equations

In this appendix we give the explicit expressions for $\mathcal{S}$ and the coefficients a, b, and d, entering the definition of $\check{\phi}$. We use the shorthand notation $\cosh\theta_1 = c_1$, $\sinh\theta_1 = s_1$, $\cos\theta_2 = c_2$, and $\sin\theta_2 = s_2$.

We begin with the three coefficients which give the matrix $\check{\phi}$.

Knowing θ from equation (20), we first obtain:

$$a = \frac{-2i\varepsilon g_D c^2 c_1 s_2 + g_S c^2 c_1 \langle Z_S A_S\rangle + g_N \langle Z_N A_N\rangle}{B} \quad (38)$$

with

$$\begin{aligned} A_S &= 2\,c_2 + \Gamma_S\left(2\,c_1 c_2 Z_S^* + Z_S c_1 c_2 + i Z_S s_2 s_1\right) s_2 \\ &\quad - {\Gamma_S}^2 |Z_S|^2 c_1 {c_2}^2 \cosh\theta\,, \qquad (39)\\ A_N &= 2\,c^2 c_1 s_2 - 4\,c c_1 s_2 - 2i\sinh\theta - 2\,\Gamma_N Z_N^* c_2 s_2 \\ &\quad \times(-s_1 - c_1 + c_1 c)\,(s_1 - c_1 + c_1 c) \\ &\quad + i\sinh\theta\,\Gamma_N Z_N\left(\cosh\theta + c_1 c_2 c^2 - 2\,c_1 c_2 c\right) \\ &\quad + i\sinh\theta\,{\Gamma_N}^2 |Z_N|^2\,(s_1 - c_1 + c_1 c) \\ &\quad \times(-s_1 - c_1 + c_1 c)\,{s_2}^2, \qquad (40)\\ B &= g_N\langle 2\,i Z_N\cosh\theta - i{Z_N}^2\Gamma_N \sinh^2\theta\rangle \\ &\quad + g_S\langle i{Z_S}^2\Gamma_S\cosh^2\theta - 2\,Z_S\sinh\theta\rangle + 2\,g_D\varepsilon\cosh\theta. \qquad (41)\end{aligned}$$

Then

$$b = c\,(1 - 2f_{L0}^2) + f_{T0}^2 b_T \quad (42)$$

with

$$\begin{aligned} b_T &= -2c^2 - \frac{1}{8\varepsilon g_D c_1 s_2} \\ &\quad \times\langle |Z_N|^4\left[\beta_0 + \Gamma_N\beta_1 + \Gamma_N^2\beta_2 + \Gamma_N^3\beta_3 + \Gamma_N^4\beta_4\right]\rangle \qquad (43)\end{aligned}$$

with

$$\begin{aligned} \beta_0 &= 16\,c_1 g_N c_2 a_1 + 16\,\varepsilon\,g_D c_1 c c_2 a_2 - 16\,s_2 g_N s_1 a_2\,, \qquad (44)\\ \beta_1 &= -8\,c_1 g_N s_1\left(-1 + 2\,{c_2}^2\right)(c-1)^2 \\ &\quad + 8\,a_1 g_N\left(2\,{c_1}^2 - 1 - 3\,c_1 c_2 + 2\,{c_2}^2\right) \\ &\quad + a_2\left(32\,\varepsilon\,g_D {c_1}^2 c {c_2}^2 - 32\,g_D\varepsilon\,c_1 c c_2 + 24\,s_2 g_N s_1\right)\,, \qquad (45)\\ \beta_2 &= -8\,c_1 g_N s_1\,(c-1)^2\left(1 + c_1 c_2 - 2\,{c_2}^2\right) \\ &\quad + 4\,a_1 g_N\left(4\,c_1 c_2 + c_1 {c_2}^3 - 4\,{c_2}^2 - 4\,{c_1}^2 + 2 + {c_1}^3 c_2\right) \\ &\quad + a_2\big(8\,c\varepsilon\,g_D c_1 c_2\left(2\,{c_1}^2{c_2}^2 + {c_1}^2 + 2 + {c_2}^2 - 6\,c_1 c_2\right) \\ &\quad + 4\,s_2 g_N s_1\left({c_1}^2 - 2 + {c_2}^2\right)\big)\,, \qquad (46)\\ \beta_3 &= -2\,(c_1 - c_2)^2 c_1 g_N s_1\,(c-1)^2 - 2\,a_1\,(c_1 - c_2)^2 \\ &\quad \times g_N\,(c_1 c_2 - 1) \\ &\quad + 2\,a_2\big(4\,c\,(c_1 - c_2)^2\,\varepsilon\,g_D c_1 c_2\,(c_1 c_2 - 1) \\ &\quad - (c_1 - c_2)^2\,s_2 g_N s_1\big)\,, \qquad (47)\\ \beta_4 &= \varepsilon\,g_D c_1 c c_2 a_2\,(c_1 - c_2)^4\,. \qquad (48)\end{aligned}$$

Finally

$$d = -2f_{L0}f_{T0}\left(1 + c^2 + a_2\tan\theta_2\right)\,. \quad (49)$$

The explicit form of $\mathcal{S}_T(E)$ reads:

$$\mathcal{S}_T = \frac{g_S c_1}{16}\langle |Z_S|^4\left[\alpha_0 + \Gamma_S\alpha_1 + \Gamma_S^2\alpha_2 + \Gamma_S^3\alpha_3\right]\rangle \quad (50)$$

with

$$\alpha_0 = -16\,c^3 s_2 - 8\,b_T s_2 - 16\,c c_2 a_2\,, \quad (51)$$

$$\alpha_1 = 8\,a_1 c s_1 \left(-1 + 2\,{c_2}^2\right) + 24\,c c_2 a_2 + 16\,c^3 c_1 + 24\,c^3 s_2 + 12\,b_T c_1 - 8\,b_T c_1 {c_2}^2 + 12\,b_T s_2\,, \quad (52)$$

$$\alpha_2 = 8\,a_1 c s_1 \left(-2\,{c_2}^2 + 1 + c_1 s_2\right) + 4\,a_2 c c_2 \left({c_1}^2 - 1 - {c_2}^2\right) - 4\,c^3 \left(4\,c_1 - s_2 {c_2}^2 + 3\,s_2 + s_2 {c_1}^2\right) + b_T \left(-12\,c_1 + 8\,c_1 {c_2}^2 - 6\,s_2 + 2\,s_2 {c_2}^2 - 6\,s_2 {c_1}^2\right)\,, \quad (53)$$

$$\alpha_3 = -2\,a_1 c s_1 \left(-{c_2}^2 + {c_1}^2 + 1 + 2\,c_1 s_2\right) - 2\,a_2 c c_2 \left(-{c_2}^2 + {c_1}^2 + 1 + 2\,c_1 s_2\right) + 2\,c^3 \left(s_2 + 2\,c_1 + s_2 {c_1}^2 - 2\,c_1 {c_2}^2 - s_2 {c_2}^2\right) + b_T \left(-3\,c_1 {c_2}^2 + s_2 + {c_1}^3 + 3\,c_1 + 3\,s_2 {c_1}^2 - s_2 {c_2}^2\right)\,. \quad (54)$$

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